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2705 [May, 1918]. Proposed by PAUL CAPRON, U. S. Naval Academy.

Find the area of a loop of the trochoid

$$x = \frac{1}{3}a(3\phi - \pi \sin \phi), \quad y = \frac{1}{3}a(3 - \pi \cos \phi).$$

SOLUTION BY L. E. MENSENKAMP, Freeport, Illinois.

The curve of the type here given is generated by causing a circle of radius a to roll on a straight line. Then, a point on a fixed radius of this circle, and at a distance $\pi a/3$ from the center, will describe the given trochoid. The axis of X is the line on which the circle rolls, and the axis of Y is taken as the position of the radius containing the generating point when it is pointing vertically downward.

The desired area may be found by evaluating $\int y dx$ between the proper limits of integration. It is convenient to express this integral in terms of the parameter ϕ , as

$$\int \frac{a^2}{9} (3 - \pi \cos \phi)^2 d\phi.$$

From the way in which the curve is defined above, it is symmetrical with respect to the Y -axis; therefore, it will suffice to find that portion of the loop to the left of the Y -axis and multiply it by 2. As ϕ starts from zero and increases in value, the point which generates the curve starts from its position on the negative portion of the Y -axis and traverses the left half of the loop in a clock-wise direction about the origin, finally returning to the Y -axis again, this time in the upper half of the coördinate plane. The limits of integration in ϕ must, therefore, correspond to the two points where the loop cuts the Y -axis. Examination of the equations of the curve shows these to be 0 and $\pi/6$.

The area of half the curve is therefore given by

$$\frac{a^2}{9} \int_0^{\pi/6} (9 - 6\pi \cos \phi + \pi^2 \cos^2 \phi) d\phi,$$

or

$$\frac{a^2}{9} \left(9\phi - 6\pi \sin \phi + \frac{\pi^2 \phi}{2} + \frac{\pi^2}{4} \sin 2\phi \right)_0^{\pi/6}.$$

The area of the entire loop will then be double this, or

$$\frac{a^2 \pi}{108} (-36 + 2\pi^2 + 3\pi\sqrt{3}).$$

Also solved by J. B. REYNOLDS and the PROPOSER.

2707 [May, 1918]. Proposed by S. A. COREY, Albia, Iowa.

Let A , B , and C be the vector sides of a triangle. Construct another triangle with vector sides R , S , and T , where

$$R = mA - dnB, \quad S = nA + (m + en)B.$$

Then prove that

$$(m^2 + emn + dn^2)(a^2 + eab \cos (AB) + db^2) = r^2 + ers \cos (RS) + ds^2,$$

where d , e , m and n are any scalar quantities; a , b , r and s are the tensors, or lengths, of the sides A , B , R and S , respectively; and $\cos (RS)$ is the cosine of the angle between R and S when placed coinitially.

SOLUTION BY THE PROPOSER.

We have the algebraic identity

$$(m^2 + emn + dn^2)(a^2 + eab + db^2) = r^2 + ers + ds^2, \quad (1)$$

where $r = ma - dnB$, and $s = na + (m + en)b$.

As (1) is a quadratic identity in a , b , r , and s , it holds if these quantities be vectors (or quaternions), provided we consider the scalar part only of the vector products which we would have in (1) under such an interpretation. But such an interpretation gives us exactly the equation of the given problem, which is therefore true.